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New $AdS_3 \times G/H$ compactifications of chiral IIB supergravity

Leonardo Castellani* **and Luca Sommovigo**

Dipartimento di Scienze e Tecnologie Avanzate
Università del Piemonte Orientale, Italy
E-mail: castellani@to.infn.it, sommovigo@mfn.unipmn.it

Abstract: We present a new class of solutions of $D = 10$, $N = 2$ chiral supergravity. A non-vanishing background for the field strength G_{MNR} of the complex two-form triggers $AdS_3 \times M_7$ compactifications, where M_7 is a 7-dimensional compact manifold. When M_7 is a non-symmetric coset space G/H, we can always find a set of constants G_{MNR} covariantly conserved and thus satisfying the field equations. For example the structure constants of G with indices in the G/H directions are a covariantly conserved tensor. In some symmetric G/H, where these structure constants vanish, there may still exist conserved 3-forms, yielding $AdS_3 \times G/H$ solutions. The conditions for supersymmetry of the $AdS_3 \times M_7$ compactifications are derived, and tested for the supersymmetric solutions $AdS_3 \times S^3 \times \mathbb{T}^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$. Finally, we show that the $AdS_3 \times S^3 \times \mathbb{T}^4$ supersymmetric background can be seen as the $\sigma = 0$ limit in a one-parameter class of solutions of the form $AdS_3 \times S^3 \times \mathbb{CP}^2$, the parameter σ being the inverse "radius" of $\mathbb{CP}^2$. For $\sigma \neq 0$ all supersymmetries are broken.

Keywords: Supergravity Models, AdS-CFT Correspondance, Superstrings and Heterotic Strings, Superstring Vacua.

*Dipartimento di Fisica Teorica and Istituto Nazionale di Fisica Nucleare, Via P. Giuria 1, 10125 Torino, Italy

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1. Introduction

Spontaneous Anti de-Sitter compactifications of $D = 10$ chiral $N = 2$ supergravity have received some attention recently, in the light of the conjectured AdS/CFT correspondence [1]. In particular $AdS_5 \times M_5$ solutions [2, 3], where $M_5 =$ compact space, have been used to glean information on the related 4-dimensional conformal field theory [1, 5, 3, 4]. The presence of a (complex) three-form field strength can trigger compactifications on $AdS_3 \times M_7$, an example being the known solutions $M_7 = S^3 \times \mathbb{T}^4$ and $M_7 = S^3 \times S^3 \times S^1$, whose corresponding two-dimensional conformal theory has been investigated in [6]. Further refs on IIB compactifications on spheres can be found in [7].

In this letter we present a general class of $AdS_3 \times M_7$ solutions with $M_7 =$ compact 7-dimensional coset space G/H, and derive the conditions for these solutions to be supersymmetric backgrounds. This class of compactifications may be relevant for the study of the two-dimensional CFT's examined in ref. [8].

The theory contains a complex anti-Weyl gravitino ψ_M and a complex Weyl spinor λ. The bosonic fields are: the graviton g_{MN}, a complex antisymmetric tensor A_{MN}, a real antisymmetric tensor A_{MNRS} (restricted by a self-duality condition) and a complex scalar ϕ. There is a global U(1) symmetry that rotates the two supersymmetry charges into each other. According to a general mechanism in supergravity theories, the scalars can be interpreted as coordinates of non-compact coset spaces. Here the complex scalar ϕ parametrizes the coset $\mathrm{SU}(1,1)/\mathrm{U}(1)$.

After setting the spinor fields to zero, the field equations read [9, 10, 11]:

$$\begin{aligned} 2R_{MN} &= P_M P^*_N + P^*_M P_N + \frac{1}{6} F^{PQRS}{}_M F_{PQRSN} + \\ &\quad + \frac{1}{8}\left(G^{PQ}{}_M G^*_{PQN} + G^{*\ PQ}{}_M G_{PQN} - \frac{1}{6} g_{MN} G^{PQR} G^*_{PQR} \right), \end{aligned} \tag{1.1}$$

$$F_{M_1-M_5} = \frac{1}{5!}\epsilon_{M_1-M_5 N_1-N_5} F^{N_1-N_5}\,, \tag{1.2}$$

$$(\nabla^S - iQ^S) G_{MNS} = P^S G^*_{MNS} - \frac{2i}{3} F_{MNPQR} G^{PQR}\,, \tag{1.3}$$

$$(\nabla^M - 2iQ^M) P_M = -\frac{1}{24} G^{PQR} G_{PQR}\,, \tag{1.4}$$

where $R_{MN} \equiv R^S{}_{M\ SN}$ and the curvature two-form is defined as $R^S{}_M \equiv dB^S{}_M + B^S{}_N \wedge B^N{}_M$; the vectorial quantities P_M (complex) and Q_M (real) are related to the scalar fields (and derivatives thereof), F_{PQRSN} and G_{PQN} to the field strengths of the four-form and of the two-form, and ∇ is the Lorenz covariant derivative. Here we have adopted the normalizations and conventions of [9]; note however a sign correction in (1.4), already found in [11] and noted also in [4]. Moreover the following Bianchi identities hold (a consequence of the field definitions):

$$(\nabla_{[M} - 2iQ_{[M}) P_{N]} = 0\,, \qquad \partial_{[M} Q_{N]} = -iP_{[M} P^*_{N]}\,, \tag{1.5}$$

$$(\nabla_{[M} - iQ_{[M}) G_{NRS]} = -P_{[M} G^*_{NRS]}\,, \tag{1.6}$$

$$\partial_{[N} F_{M_1 \ldots M_5]} = \frac{1}{8}\,\mathrm{Im}\, G_{[NM_1M_2} G^*_{M_3M_4M_5]}\,. \tag{1.7}$$

The supersymmetry variations of the bosonic fields are proportional to Fermi fields, and these vanish in the type of backgrounds we are considering. On the other hand, the supersymmetry variations of the fermionic fields are:

$$\delta\lambda = i\Gamma^M \varepsilon^* P_M - \frac{1}{24} i G_{MNP} \Gamma^{MNP} \varepsilon\,, \tag{1.8}$$

$$\begin{aligned} \delta\psi_M &= \left(\nabla_M - \frac{i}{2} Q_M\right)\varepsilon + \frac{i}{480} F_{N_1-N_5}\Gamma^{N_1-N_5}\Gamma_M \varepsilon + \\ &\quad + \frac{1}{96}\left(\Gamma_M{}^{N_1-N_3} G_{N_1-N_3} - 9\Gamma^{N_1N_2} G_{MN_1N_2}\right)\varepsilon^* \end{aligned} \tag{1.9}$$

(in backgrounds with $\psi = 0, \lambda = 0$) [9]. A solution is supersymmetric if there exist spinors ε for which these variations vanish.

2. The ansatz for the $AdS_3 \times M_7$ solutions

We use the index conventions

M,N,P...= 1-10

m,n,p...= 1-3 (run on AdS_3)

a,b,c...= 4-10 (run on M_7).

and the flat "mostly minus" D=10 metric $\eta = (+,-,-;-,-,-,-,-,-,-)$. With the Ansatz:

$$\begin{aligned} g_{MN} &= \text{metric of } AdS_3 \times M_7 \,, \\ \text{fermions} &= 0\,; \qquad P_M = Q_M = F_{M_1-M_5} = 0\,, \\ G_{mnp} &= e\epsilon_{mnp}\,, \qquad G_{abc} = gJ_{abc}\,, \end{aligned} \tag{2.1}$$

where e and g are complex constants and J_{abc} a real constant antisymmetric tensor, the field eqs. (1.4) take the form:

$$6e^2 - g^2J^2 = 0\,, \quad J^2 \equiv J_{abc}J_{abc} > 0 \tag{2.2}$$

implying

$$g = \rho\frac{\sqrt{6}}{J}e\,, \quad \rho = \pm 1\,. \tag{2.3}$$

Substituting the Ansatz (2.1)and the relation (2.3) into the remaining field equations yields:

$$R_{mn} = \frac{1}{4}|e|^2 g_{mn}\,, \tag{2.4}$$

$$R_{ab} = \frac{3}{4}\frac{J_{ab}}{J^2}\,|e|^2\,, \quad J_{ab} \equiv J_{acd}J_{bcd}\,, \tag{2.5}$$

$$\nabla^c J_{abc} = 0\,, \quad \nabla = M_7 - \text{covariant derivative}\,. \tag{2.6}$$

Equation (1.2) is trivially satisfied, while eq. (1.3) with free indices m, n holds because ϵ_{mnp} is a covariantly conserved tensor in AdS_3. Moreover it is immediate to check that the Bianchi identities (1.5)–(1.7) are satisfied. Thus our Ansatz is a solution of the classical 2b supergravity equations provided eqs. (2.4)–(2.6) hold. The first equation fixes the AdS_3 radius. If there exist an M_7 — covariantly conserved three-index antisymmetric tensor J_{abc} in M_7 the third equation is satisfied, and the second equation becomes a condition on the Ricci tensor of M_7.

As we show in the following, such J_{abc} always exist in non-symmetric coset spaces G/H (and in various symmetric G/H). Moreover J_{ab} is diagonal, allowing in most cases G/H to solve eq. (2.5) after a vielbein rescaling.

3. G/H geometry and the tensor J_{abc}

The structure constants of $\mathbb{G} = \mathbb{H} + \mathbb{K}$ are defined by

$$
\begin{aligned}
&[H_i, H_j] = C_{ij}{}^k H_k\,, \quad H_i \in \mathbb{H}\,,\\
&[H_i, K_a] = C_{ia}{}^j H_j + C_{ia}{}^b K_b\,, \quad K_a \in \mathbb{K}\,,\\
&[K_a, K_b] = C_{ab}{}^j H_j + C_{ab}{}^c K_c\,,
\end{aligned}
\tag{3.1}
$$

where the index conventions are obvious. As discussed in ref. [12, p. 251], whenever H is compact or semisimple one can always find a basis of K_a such that the structure constants $C_{ia}{}^j$ vanish. In that case the $\mathbb{G} = \mathbb{H} + \mathbb{K}$ split, or equivalently the coset space G/H is said to be *reductive*. For this reason we will deal in this paper only with reductive coset spaces. Another important observation is that when G/H is reductive the structure constants $C_{ia}{}^b$ can always be made antisymmetric in a, b by an appropriate redefinition $K_a \to N_a{}^b K_b$ [12, 13].

For later use we recall the expression of the G/H riemannian connection

$$
B^a{}_b = \frac{1}{2}\left(-\frac{r_b r_c}{r_a} C_{bc}{}^a + \frac{r_a r_c}{r_b}\eta_{bg} C_{dc}{}^g \eta^{ad} + \frac{r_a r_b}{r_c}\eta_{cg} C_{db}{}^g \eta^{ad}\right) V^c - C_{bi}{}^a \Omega^i\,, \tag{3.2}
$$

where V^c and Ω^i are the K and H vielbeins, respectively, and we have allowed for rescalings r_c of V^c in the isotropy irreducible subspaces of K, see refs. [14, 15, 16]. These subspaces correspond to the block-diagonal pieces of the matrices $C_{bi}{}^a$, so that

$$
\frac{r_a}{r_b}\, C_{ia}{}^b = C_{ia}{}^b\,. \tag{3.3}
$$

The G/H Riemann curvature is defined by $R^a{}_b \equiv dB^a{}_b + B^a{}_c \wedge B^c{}_b \equiv R^a{}_{b\,de} V^d \wedge V^e$ and reads [16]:

$$
R^a{}_{b\,de} = \frac{1}{4}\frac{r_d r_e}{r_c}\bar{C}_{bc}{}^a C_{de}{}^c + \frac{1}{2} r_d r_e\, C_{bi}{}^a C_{de}{}^i + \frac{1}{8}\bar{C}_{cd}{}^a \bar{C}_{be}{}^c - \frac{1}{8}\bar{C}_{ce}{}^a \bar{C}_{bd}{}^c \tag{3.4}
$$

with

$$
\bar{C}_{bc}{}^a \equiv \frac{r_b r_c}{r_a}\, C_{bc}{}^a - \frac{r_a r_c}{r_b}\, C_{ac}{}^b - \frac{r_a r_b}{r_c}\, C_{ab}{}^c\,. \tag{3.5}
$$

Consider now the field eq. (2.6), i.e.:

$$
B_e{}^d{}_{[a} J_{bc]d}\, \eta^{ec} = 0\,, \tag{3.6}
$$

where the connection 1-form components are defined by $B^d{}_a \equiv B_c{}^d{}_a V^c$. One possible choice for a J_{abc} satisfying (3.6) is given by:

$$
J_{abc} = C_{abc} \equiv C_{ab}{}^G \gamma_{cG}\,, \quad \text{G runs on the group } G,\ \gamma = \text{Killing metric}\,. \tag{3.7}
$$

Indeed $B_e{}^d{}_{[a} C_{bc]d}\, \eta^{ec} = 0$ holds for the following reason. Observe that the left hand side is an H-invariant tensor, since the connection components, the structure constants C_{abc} and the Killing metric are all H-invariant tensors. By H-invariant tensor we mean, for example, that:

$$\delta B_c{}^a{}_b \equiv C_{ic}{}^{d} B_d{}^a{}_b - C_{id}{}^{a} B_c{}^d{}_b + C_{ib}{}^{d} B_c{}^a{}_d = 0\,, \tag{3.8}$$

i.e. the adjoint action of H on B vanishes. It is not difficult to prove that in (3.2) the term multiplying V^c is H-invariant. In fact each of the three terms within parentheses in (3.2) is H-invariant, as one can show by using Jacobi identities and (3.3).

In general the only H-invariant tensor with two free indices is the Killing metric (except in the special case of S^2, where one has also ϵ_{ab}), and therefore $B_e{}^d{}_{[a} C_{bc]d}\,\eta^{ec}$, being antisymmetric in its free indices, has to vanish.

In conclusion, the choice (3.7) satisfies the field eqs. (2.6). Moreover $J_{ab} \equiv J_{acd} J_{bcd}$, being a symmetric H-invariant tensor, must be proportional to the Killing metric in the H-isotropy subspaces.

The structure constants C_{abc} are not the most general solution to eq. (3.6). For example also Antisymm $(C_{ab}{}^c)$, i.e. the antisymmetrization of $C_{ab}{}^c$ on its three indices, satisfies eq. (3.6). In fact there may exist J_{abc} tensors satisfying (3.6) even for symmetric G/H; this happens obviously for $S^3 = \mathrm{SO}(4)/\,\mathrm{SO}(3)$, where J_{abc} is proportional to ϵ_{abc}, or less trivially in the case $G/H = S^3 \times \mathbb{CP}^2$ discussed in section 7.

4. Supersymmetry conditions

We adopt the following real representation of the $D = 3+7$ gamma matrices:

$$\mathbf{\Gamma}_M = (\gamma_m \otimes 1\!\mathrm{I}_{8\times 8} \otimes \sigma_2\,,\ 1\!\mathrm{I}_{2\times 2} \otimes \Gamma_a \otimes \sigma_1)\,, \tag{4.1}$$

where the $\mathrm{SO}(1,2)$ gamma matrices are:

$$\gamma_1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \gamma_2 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \qquad \gamma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \tag{4.2}$$

and the real $\mathrm{SO}(7)$ gamma matrices are given by the octonion structure constants (totally antisymmetric):

$$(\Gamma_a)_{bc} = a_{abc}\,, \qquad (\Gamma_a)_{b8} = \delta_{ab}\,, \tag{4.3}$$

$$a_{abc}:\ [123] = [165] = [257] = [354] = [367] = [246] = [147] = 1\,. \tag{4.4}$$

The supersymmetry parameter ε has the same Weyl chirality as the gravitino ψ, i.e. $\mathbf{\Gamma}_{11}\varepsilon = -\varepsilon$ (anti-Weyl). Any anti-Weyl $\mathrm{SO}(1,9)$ spinor can be decomposed as:

$$\varepsilon = c_N \xi^N \otimes \eta^N \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{4.5}$$

where ξ^N and η^N are real SO(1,2) and SO(7) spinors, respectively, and $c_N \in \mathbb{C}$. Substituting in the λ supersymmetry condition:

$$G_{MNR}\mathbf{\Gamma}^{MNR}\varepsilon = 0 \Longrightarrow (e\epsilon_{mnr}\mathbf{\Gamma}^{mnr} + gJ_{abc}\mathbf{\Gamma}^{abc})\varepsilon = 0 \tag{4.6}$$

yields:

$$c_N\xi^N \otimes (6e\mathbb{1} + gJ_{abc}\Gamma^{abc})\eta^N = 0\,, \tag{4.7}$$

which can hold only if there exist SO(7) spinors satisfying:

$$\left(\mathbb{1} + \rho\frac{J_{abc}}{\sqrt{6}J}\Gamma^{abc}\right)\eta = 0\,, \tag{4.8}$$

where we have also used (2.3).

Consider now the ψ supersymmetry condition:

$$\left(d + \frac{1}{4}B^{mn}\mathbf{\Gamma}_{mn} + \frac{1}{4}B^{ab}\mathbf{\Gamma}_{ab}\right)\varepsilon + \frac{1}{96}V^m\left(G_{abc}\mathbf{\Gamma}_m\mathbf{\Gamma}^{abc} - 9e\epsilon_{mnr}\mathbf{\Gamma}^{nr}\right)\varepsilon^* +$$
$$+\frac{1}{96}V^d\left(G_{abc}\mathbf{\Gamma}_d{}^{abc} + e\epsilon_{mnr}\mathbf{\Gamma}_d\mathbf{\Gamma}^{mnr} - 9G_{dab}\mathbf{\Gamma}^{ab}\right)\varepsilon^* = 0 \tag{4.9}$$

V^m and V^d are the AdS_3 and M_7 vielbeins, respectively. Substituting the $\mathbf{\Gamma}$-matrix and ε decomposition leads to:

$$c\left(\partial_m\xi + \frac{1}{4}B^{rs}{}_m\gamma_{rs}\xi\right) - \frac{1}{4}ic^*e\gamma_m\xi = 0\,, \tag{4.10}$$

$$c\left(\partial_c\eta + \frac{1}{4}B^{ab}{}_c\Gamma_{ab}\eta\right) - \frac{1}{8}c^*G_{cab}\Gamma^{ab}\eta = 0\,, \tag{4.11}$$

where we have dropped the index N since the conditions can be satisfied only if they hold separately for every N. Moreover use of (4.8) has been necessary in order to achieve the factorization of (4.9) into (4.10) and (4.11). The modulus of c is irrelevant in (4.10) and (4.11), and we can set $c = \exp(i\varphi)$. Then the integrability condition for (4.10) is:

$$\left[\frac{1}{4}R^{mn}{}_{rs}\gamma_{mn} - \left(\frac{1}{4}\right)^2\exp(-4i\varphi)e^2\gamma_{rs}\right]\xi = 0\,. \tag{4.12}$$

The field equations (2.4) tell us that the AdS_3 curvature is:

$$R^{mn}{}_{rs} = \frac{1}{4}|e|^2\delta^{mn}_{rs} \tag{4.13}$$

so that the integrability condition can be satisfied if and only if the phase of c is such that:

$$|e|^2 = e^2\exp(-4i\varphi) \Longrightarrow e = \alpha|e|\exp(2i\varphi)\,, \quad \alpha = \pm 1 \tag{4.14}$$

cf. ref. [17]. Using this relation and (2.3) into (4.11) yields finally:

$$\partial_m \xi + \frac{1}{4} B^{rs}{}_m \gamma_{rs} \xi - \frac{1}{4} i\alpha |e| \gamma_m \xi = 0\,, \tag{4.15}$$

$$\partial_c \eta + \frac{1}{4} B^{ab}{}_c \Gamma_{ab} \eta - \frac{\sqrt{6}}{8} \alpha\rho |e| \frac{J_{cab}}{J} \Gamma^{ab} \eta = 0\,. \tag{4.16}$$

The integrability condition for (4.16) reads:

$$\left(R^{ab}{}_{cd} + \frac{3}{2} |e|^2 \frac{J_{cae} J_{dbe}}{J^2} \right) \Gamma_{ab} \eta = 0\,. \tag{4.17}$$

The $AdS_3 \times M_7$ solution preserves N supersymmetries if and only if there exist N independent SO(7) spinors η satisfying simultaneously eqs. (4.8) and (4.16).

In next Section we test our formulae in the case of the two known supersymmetric solutions, corresponding to $G/H = S^3 \times \mathbb{T}^4$ and $G/H = S^3 \times S^3 \times S^1$.

5. The supersymmetric solutions $AdS_3 \times S^3 \times \mathbb{T}^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$

We'll treat the two solutions simultaneously. The J_{abc} tensors are simply the ϵ_{abc} Levi-Civita tensors in the S^3 directions, so that Einstein field equations (2.5) are respectively:

$$\begin{aligned} &R_{ab} = \frac{1}{4} |e|^2 \delta_{ab}\,, \quad a,b = 1,2,3\,, \\ &R_{ab} = 0\,, \quad a,b = 4,5,6,7 \end{aligned} \tag{5.1}$$

and

$$\begin{aligned} &R_{ab} = \frac{1}{8} |e|^2 \delta_{ab}\,, \quad a,b = 1,2,\ldots 6\,, \\ &R_{ab} = 0\,, \quad a,b = 7 \end{aligned} \tag{5.2}$$

fixing the radii of the S^3 spheres.

In the real gamma matrix representation of section 4, the $\delta\lambda = 0$ supersymmetry condition is satisfied by any linear combination of the four independent spinors $\eta_1, \eta_2, \eta_3, \eta_8$, the 8-dimensional spinor η_a having the a-th component as only non-vanishing component. This holds for both solutions. On the other hand, the $\delta\psi = 0$ supersymmetry condition is satisfied by 8 independent real spinors $\eta_1^\pm, \eta_2^\pm, \eta_3^\pm, \eta_8^\pm$, the $^\pm$ referring to the sign α of (4.16). These spinors depend on the M_7 coordinates. Again this holds for both solutions. The two compactifications have then $N = 8$ supersymmetries, or 16 real conserved supercharges (since ξ has two real components).

6. Other $AdS_3 \times G/H$ solutions

A complete classification of all $AdS_3 \times G/H$ solutions based on the Ansatz (2.1) is postponed to a later publication. This requires to find the most general J_{abc} that solves eq. (3.6) for each 7-dimensional G/H.

Here we give some selected examples, choosing some particular J tensors. In fact, all the 7-dimensional G/H cosets classified in [18] are solutions of the IIB field equations, after suitable rescalings of the coset vielbeins. But the G/H list of IIB solutions is actually larger than the one relevant for $D = 11$ supergravity compactified on $AdS_4 \times G/H$. Indeed in the IIB case the field equations do not force G/H to be an Einstein space, but rather to be "isotropy Einstein", i.e. with a Ricci tensor R_{ab} proportional to δ_{ab} in each isotropy irreducible subspace of G/H. The proportionality constant can also vanish: when this happens in one-dimensional subspaces S^1 factors are allowed (they were excluded in the list of [18]).

6.1 $AdS_3 \times M^{pqr}$

The M^{pqr} spaces have been studied in detail in refs. [19, 15, 22]. They have three isotropy irreducible subspaces, allowing three independent rescalings a, b, c of the vielbeins corresponding to the (1,2), 3, (4,5,6,7) directions and preserving the $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ isometry. Their Ricci tensor is given by [19]:

$$\begin{aligned} R_{mn} &= \frac{1}{4} b^2 \left(2 - \frac{b^2}{c^2} q^2 \right) \delta_{mn} \,, \quad m, n = 1, 2 \,, \\ R_{33} &= \frac{9a^4 p^2 + 2b^4 q^2}{8c^2} \,, \\ R_{AB} &= \frac{3}{16} a^2 \left(4 - 3 \frac{a^2}{c^2} p^2 \right) \delta_{AB} \,, \quad A, B = 4, 5, 6, 7 \,. \end{aligned} \tag{6.1}$$

For $q \neq 0$, $p \neq 0$ we can redefine:

$$a = \frac{q}{p} \gamma \sqrt{\frac{2\alpha}{3}} \,, \qquad b = \gamma \sqrt{2\beta} \,, \qquad c = q\gamma \tag{6.2}$$

so that the Ricci tensor becomes:

$$\begin{aligned} R_{mn} &= \gamma^2 \beta (1 - \beta) \delta_{mn} \,, \quad m, n = 1, 2 \,, \\ R_{33} &= \gamma^2 \left(\beta^2 + \frac{1}{2} \frac{q^2}{p^2} \alpha^2 \right) , \\ R_{AB} &= \frac{1}{2} \gamma^2 \alpha \left(1 - \frac{1}{2} \alpha \right) \frac{q^2}{p^2} \delta_{AB} \,. \end{aligned} \tag{6.3}$$

An explicit check reveals that taking J_{abc} to be the Levi-Civita tensor in the directions 1,2,3 (as in the case of the $S^3 \times \mathbb{T}^4$ solution) and otherwise zero satisfies the condition (3.6). Then the field equations are as in (5.1), and are satisfied by the Ricci tensor in (6.3) when the rescalings are:

$$\alpha = 2\,, \qquad \beta = \frac{1 \pm \sqrt{1 - 16\frac{q^2}{p^2}}}{4}\,, \qquad \gamma = \frac{1}{4\beta(1-\beta)}|e|^2 \tag{6.4}$$

requiring $p/q \geq 4$. The particular case $q = 0$, corresponding to $S^2 \times S^5$, is also a solution: the rescalings are then:

$$a^2 = \frac{1}{6}|e|^2\,, \qquad b^2 = \frac{1}{2}|e|^2\,, \qquad c^2 = \frac{1}{8}p^2|e|^2\,. \tag{6.5}$$

The $\delta\lambda$ supersymmetry condition is satisfied by the same spinors $\eta_1, \eta_2, \eta_3, \eta_8$ discussed in the previous Section. However these spinors do not satisfy the $\delta\psi$ condition, and therefore these solutions are not supersymmetric.

There are other possible choices for J_{abc}. For example $J_{abc} = \text{Antisymm}(C_{ab}{}^{c})$ leads to field equations that can still be solved by a set of different rescalings α, β, γ. In this case the four directions 4,5,6,7 are not Ricci flat.

6.2 $AdS_3 \times N^{010}$

The N^{010} coset spaces are a special case in the class of N^{pqr} coset spaces studied in refs. [20, 21, 16, 23] in the context of $D = 11$ supergravity compactifications. They can be realized as the quotient:

$$N^{010} = \frac{\text{SU}(3) \times \text{SU}(2)}{\text{SU}(2) \times \text{U}(1)}\,, \tag{6.6}$$

where the SU(2) in the denominator is diagonally embedded in $G = \text{SU}(3) \times \text{SU}(2)$. In this formulation the full isometry of N^{010} comes from the left action of G [20, 16]. The N^{010} geometry has been studied in detail in [16], and the Ricci tensor is given by:

$$R_{ab} = \left(\alpha^2 + \frac{1}{32}\frac{\beta^4}{\alpha^2}\right)\delta_{ab}\,, \tag{6.7}$$

$$R_{AB} = \frac{3}{4}\beta^2\left(1 - \frac{1}{16}\frac{\beta^2}{\alpha^2}\right)\delta_{AB}\,. \tag{6.8}$$

Again one can check explicitly that the same $J_{abc} = \epsilon_{abc}, a, b, c = 1, 2, 3$ used in the previous solutions satisfies the field equations (3.6). Then $AdS_3 \times N^{010}$ is a solution provided the rescalings are fixed to the values:

$$\alpha = \pm\frac{1}{6}|e|\,, \qquad \beta = \pm\frac{2}{3}|e|\,. \tag{6.9}$$

As in the previous case supersymmetry is absent because (4.16) has no solutions.

In a similar way we easily find that for every 7-dimensional coset space G/H of the classification [18] there exist a set of constants J_{abc} such that $AdS_3 \times G/H$ is a solution for IIB supergravity, with the exception of the round 7-sphere SO(8)/ SO(7). Also, every $G/H \times S^1$ space with G/H = non-symmetric 6-dimensional coset is a solution of the IIB equations for appropriate rescalings and $J_{abc} = C_{abc}$.

What remains to be done is to determine in each case the most general J_{abc}, and check whether there are instances in which both supersymmetry conditions can be satisfied simultaneously, yielding supersymmetric solutions as in section 5.

We expect that, in order to make contact with the $d = 2$ superconformal theories discussed in [8], we need to extend our Ansatz to a non-constant scalar field.

7. Asymptotic supersymmetry: $S^3 \times (\mathbb{CP}^2 \to \mathbb{T}^4)$

We study here a particular class of solutions, characterized by a continuous parameter σ. Consider the cosets $G/H = M^{010}$, a special case of the M^{pqr} class with $S^3 \times \mathbb{CP}^2$ topology [19]. In general a rescaling of the G structure constants given by:

$$C_{G_1 G_2}{}^{G_3} \longrightarrow \frac{r_{G_1} r_{G_2}}{r_{G_3}} C_{G_1 G_2}{}^{G_3} \tag{7.1}$$

still defines a Lie algebra. Take all $r = 1$ except those in the $4, 5, 6, 7$ coset directions, for which $r = \sigma$. Then the Ricci tensor becomes:

$$\begin{aligned} R_{mn} &= \frac{1}{4} b^2 \left(2 - \frac{b^2}{c^2}\right) \delta_{mn} , \quad m, n = 1, 2 , \\ R_{33} &= \frac{b^4}{4c^2} , \\ R_{AB} &= \frac{3}{4} a^2 \sigma^2 \delta_{AB} , \quad A, B = 4, 5, 6, 7 . \end{aligned} \tag{7.2}$$

The limit $\sigma \to 0$ corresponds to a group contraction yielding $S^3 \times \mathbb{T}^4$: in fact it amounts to sending the radius of $\mathbb{CP}^2$ to infinity. For any σ, i.e. for any $\mathbb{CP}^2$-radius, $AdS_3 \times S^3 \times \mathbb{CP}^2$ is a solution of the IIB equations if we choose the J_{abc} tensor to be:

$$J_{123} = 1 , \qquad J_{345} = J_{367} = \sigma \tag{7.3}$$

and the rescalings:

$$a^2 = \frac{|e|^2}{3(1 + 2\sigma^2)} , \qquad b^2 = \pm c|e| , \quad c = \frac{\sigma^2 + 1}{2\sigma^2 + 1} |e| . \tag{7.4}$$

One can see easily that the $\delta\lambda$ supersymmetry condition is satisfied by just one of these solutions, corresponding to $\sigma = 0$, i.e. the $N = 8$ supersymmetric $AdS_3 \times S^3 \times \mathbb{T}^4$ compactification. All the other values of σ break supersymmetry. Thus we obtain a class of continuously connected solutions, parametrized by σ, all of them non supersymmetric except in the limit $\sigma = 0$. For $\sigma \neq 0$, S^3 is a squashed three-sphere, cf. eqs. (7.2).

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